* ** **

Abstract

*

1 Introduction

2 Literature review

leverage e¤ect

between inventories and prices

leverage *crude oil spot market*

3.1 Descriptive statistics

Q

Descriptive Statistics and P-values for the daily futures returns and realised variance for daily futures returns and realized variance 2001-2016

0:00	0:00	1:44	1:35	0:12	0:08	21:14
0:00	0:01	2:38	2:42	0:32	0:36	8:71
0:00	0:00	1:64	1:73	0:23	0:09	7:33
0:00	0:00	0:00	0:32	0:01	12:75	258:67
0:06	0:00	0:00	5:86	0:19	19:15	495:16
0:03	0:00	0:00	2:68	0:08	20:95	648:12

Test Statistics and P-values for the daily futures returns and realised variance for daily futures returns and realised variance 2001-2016

4 Estimation method

Realized Variance *Integrated Variance*

t [***t*** ***k; t***]

$$r(t;k) = \ln F$$

$$
\begin{aligned}
dp &= d\ln(F\ ) \\
&= \sqrt{V}\, dW_1 + x dPoisson(\ t) \\
dV &= (\quad V)\, dt + \sqrt{V}\, dW_2
\end{aligned}
$$

$$E(dW_1\, dW_2) = \ dt$$

$$x \quad N \ 0;$$

leverage e¤ect

inverse leverage e¤ect

4.4 Stochastic Volatility model (SV)

= 0; = 0 = 0)

$\boldsymbol{e}_1$
$\boldsymbol{e}_2$

$\boldsymbol{J}$

GMM estimates for the SV, SVJ, SVL, SVJL models for the S&P500 futures: 09/2001–06/2016

	1:507***	0:0869***	0:0424***	0:227***
	(5:87)	(4:58)	(2:92)	(29:99)
	0:00398***	0:00994***	0:00649***	0:00376***
	(8:41)	(12:56)	(5:55)	(15:69)
	0:283***	0:338***	0:249***	0:12015172***
	(6:55)	(19:62)	(17:96)	(54:75)
		0:979		0:156923006***
		(0:38)		(10:09)
		0:0159		0:038120618***
		(0:77)		(30:60)
			0:379***	0:490***
			(11:29)	(29:11)
N	3708			

*$p < 0{:}10$;** $p < 0{:}05$;*** $p < 0{:}01$

GMM estimates for the SV, SVJ, SVL, SVJL models for Natural Gas futures: 09/2001–06/2016

	0:923**	0:772***	0:760***	0:0556*
	(2:19)	(4:11)	(3:45)	(1:75)
	0:0483***	0:0568***	0:0460***	0:0545***
	(4:36)	(6:15)	(5:60)	(4:97)
	1:139**	1:041***	0:925***	0:24293***
	(2:33)	(6:23)	(3:49)	(3:82)
		0:0101***		0:04345***
		(4:03)		(18:52)
		0:932***		0:97814
		(32:63)		(0:53)
			0:201***	0:0495**
			(4:57)	(2:14)
N	3708			

*$p < 0{:}10$;** $p < 0{:}05$;*** $p < 0{:}01$

GMM estimates for the SV, SVJ, SVL models for WTI futures: 09/2001–06/2016

	0:117	0:0596*	0:0963*
	(1:43)	(1:77)	(1:71)
	0:0247***	0:0224***	0:0242***
	(5:75)	(3:45)	(6:43)
	0:176**	0:131***	0:162**
	(2:04)	(2:60)	(2:50)
		0:0190**	
		(2:44)	
		0:439***	
		(39:24)	
			0:276***
			(3:64)
N	3708		

*$p < 0.10$; ** $p < 0.05$; *** $p < 0.01$

5 Robustness check for subsamples

:440*** 0: 26:15)
(:69)

GMM estimates for S&P500, Natural Gas and WTI futures before and after September 15, 2008 (Lehman Brothers bankruptcy)

N = 1699			
	0:137***	0:0871***	0:276***
	(13:27)	(5:55)	(3:87)
	0:00331***	0:0836***	0:0328***
	(16:47)	(11:69)	(8:33)
	0:0577***	0:5455***	0:343***
	(57:58)	(12:32)	(6:36)
	0:1325***	0:0966***	
	(7:37)	(26:15)	
	0:0364***	0:4921***	
	(24:70)	(48:64)	
	0:440***	0:0137**	0:262***
	(18:40)	(2:20)	(6:66)
N = 1990			
	0:188***	0:0434	
	(13:22)	(1:46)	

6 Out-of-sample performance

	0:08014	0:19278	0:07757	0:1104
	0:26398	0:28523	0:26231	
	0:34547	0:3652	0:33651	0:378
	0:06134	0:12978	0:05884	0:07281
	0:20573	0:22121	0:20444	
	0:26411	0:28076	0:25676	0:27043

	0:00447	0:013848	0:004074	0:003308
	0:021026	0:019163	0:020501	
	0:066412	0:072497	0:057729	0:03341
	0:003215	0:005822	0:002886	0:002195
	0:015708	0:014377	0:015325	
	0:041425	0:046088	0:037161	0:023585

6.2 Diebold–Mariano test

	4:80	0:03	5:20	0:39	6:07	0:81	17:64	0:96
	0:66	0:42	0:97	0:97	1:31	1:00	12:81	1:00
	0:09	0:76	21:50	0:00	31:89	0:00	5:59	1:00
	0:05	0:83	27:44	0:00	36:50	0:00	0:67	1:00
	0:38	0:54	6:07	0:30	7:38	0:69	24:71	0:74
	0:00	0:96	1:65	0:90	2:21	0:99	9:59	1:00
	0:17	0:68	5:11	0:40	7:67	0:66	23:01	0:81
	0:27	0:61	4:30	0:51	5:96	0:82	22:44	0:84
	0:01	0:93	0:04	1:00	0:10	1:00	20:19	0:91
	0:01	0:93	0:04	1:00	0:10	1:00	18:55	0:95
	0:01	0:94	0:04	1:00	0:10	1:00	22:97	0:82
	0:01	0:93	0:04	1:00	0:10	1:00	19:40	0:93
	0:05	0:83	0:06	1:00	0:25	1:00	19:37	0:93
	0:01	0:92	0:05	1:00	0:11	1:00	13:84	0:99
	6:68	0:01	7:15	0:21	11:43	0:32	22:10	0:85
	4:05	0:04	4:37	0:50	4:29	0:93	4:76	1:00
	0:00	1:00	1:91	0:86	4:71	0:91	17:86	0:96
	0:15	0:69	0:76	0:98	1:52	1:00	15:42	0:99
	0:00	0:98	2:05	0:84	4:67	0:91	17:12	0:97
	0:10	0:75	0:65	0:99	1:30	1:00	10:38	1:00
	0:00	1:00	2:06	0:84	4:75	0:91	17:22	0:97
	0:11	0:74	0:75	0:98	1:60	1:00	10:23	1:00

0:094	0:231	3:253	0:001	65:850	0:006
0:303	0:000	7:853	0:000	37:832	0:568
0:296	0:000	8:172	0:000	40:898	0:431
0:452	0:000	9:018	0:000	34:394	0:720
0:068	0:626	0:928	0:177	85:452	0:000
0:259	0:000	7:420	0:000	43:723	0:316
0:072	0:554	0:799	0:212	87:346	0:000
0:249	0:000	7:090	0:000	39:267	0:503
0:311	0:000	8:799	0:000	6:095	1:000
0:483	0:000	9:252	0:000	0:421	1:000
0:314	0:000	8:809	0:000	5:425	1:000
0:494	0:000	9:254	0:000	0:397	1:000
0:151	0:007	7:048	0:000	20:472	0:996
0:431	0:000	9:126	0:000	1:917	1:000
0:052	0:888	2:377	0:009	34:724	0:706
0:303	0:000	8:145	0:000	21:069	0:994
0:046	0:958	1:521	0:064	42:030	0:383
0:276	0:000	7:644	0:000	34:788	0:704
0276	0				

0	0	0	0	0	0
39	31	70	196	223	166
311	319	280	154	127	184
350	350	350	350	350	350

0	0	0	0	0	0
336	45	60	165	69	304
14	305	290	185	281	46
350	350	350	350	350	350

	0	0	0	0
	71	8	258	199
	279	342	92	151
	350	350	350	350

7 Forecasting VaR and CVaR

b

Stage 1 *Backtesting the VaR and CVaR models*

Failure Rate *FR* *violation rate*

= 5% = 1%

FRVaR

FRVaR

CVaR

FRVaR *FRVaR*

$$FRVaR = \frac{1}{T} \sum_{=1} I\,(y < VaR\;)$$

$$FRVaR = \frac{1}{T} \sum_{=1} I\,(y > VaR\;)$$

VaR *VaR* *VaRs*

T $I\,(\,)$

$$Downside : I = \begin{cases} 1 & if \;\; y < VaR \\ 0 & if \;\; y \;\; VaR \end{cases}$$

$$Upside : I = \begin{cases} 1 & if \;\; y > VaR \\ 0 & if \;\; y \;\; VaR \end{cases}$$

LR

$H_0 : FR =$

LR

LR

LR

LR

b

LR

Out of sample VaR backtesting results using Simulated Volatilities at di¤erent risk levels

				VaR			LR			p values		LR	
			LR										
						LR			LR			LR	
5%	***VaR***	4:06%	7:32%	13:0%	0:69	0:24	0*	0:62	0:27	0*	0:51	0:23	0:93
	VaR	7:32%	7:32%	14:63%	0:03*	0:24	0*	0:27	0:27	0*	0:6	0:23	0:02*
1%	***VaR***	0:81%	0:81%	6:50%	0:96	0:96	0*	0:83	0:83	0*	0:9	0:9	0:52
	VaR	3:25%	4:06%	6:50%	0*	0:03	0*	0:05	0:01	0*	0:06	0:51	0:45
5%	***VaR***	13:82%	8:13%	8:13%	0*	0:14	0:14	0:01*	0:18	0:83	0*	0:13	0:30
	VaR	18:67%	9:76%	11:382%	0*	0:03*	0:01*	0:61	0:10	0:73	0*	0:02*	0:02*
1%	***VaR***	8:94%	0:81%	3:252%	0*	0:83	0:05	0:01*	0:9	0:60	0*	0:96	0:12
	VaR	13:01%	5:69%	4:878%	0*	0*	0*	0:13	0:35	0:27	0*	0*	0*
5%	***VaR***	2:44%	7:32%	8:94%	0:32	0:24	0:17	0:15	0:27	0:07	0:7	0:23	0:99
	VaR	4:88%	8:13%	11:38%	0:02*	0:13	0:01*	0:95	0:14	0*	0:21	0:18	0:63
1%	***VaR***	0:81%	0:81%	5:69%	0:96	0:96	0*	0:83	0:83	0*	0:9	0:9	0:39
	VaR	4:06%	4:06%	4:06%	0*	0:03	0:03	0:01	0:01	0:01	0:13	0:51	0:51
5%	***VaR***	2:44%		4:88%	0:32		0:7	0:15		0:95	0:6973		0:43
	VaR	8:13%		6:50%	0:02*		0:40	0:14		0:4636	0:75		0:29
1%	***VaR***	0:81%		1:63%	0:96		0:77	0:83		0:5223	0:9		0:8
	VaR	2:44%		1:63%	0:01*		0:77	0:17		0:5223	0:75		0:8

Out of sample CVaR backtesting results using Simulated Volatilities at di¤erent nominal risk levels b

b			*LR*		*p values*	*LR*		*LR*			*p values*	*LR*	*LR*
1:96%	***CVaR***	1:6%	0:8%	11:4%	0:91	0:57	0*	0:78	0:3	0*	0:8	0:9	0:57
	CVaR	3:2%	4:1%	9:8%	0*	0:26	0*	0:34	0:14	0*	0:07	0:5	0:93
0:38%	***CVaR***	0:8%	0:8%	5:7%	0:78	0:78	0*	0:5	0:5	0*	0:9	0:9	0:39
	CVaR	2:4%	2:4%	4:9%	0:04	0:042	0*	0:01	0:01	0*	0:7	0:7	0:47
1:96%	***CVaR***	0:11	0:02	0:04	0*	0:78	0:14	0:03	0:80	0:51	0*	0:92	0:26
	CVaR	0:14	0:06	0:07	0*	0:02*	0*	0:20	0:36	0:68	0*	0:03	0*
0:38%	***CVaR***	0:07	0:01	0:02	0*	0:50	0:01	0:14	0:90	0:70	0*	0:78	0:04
	CVaR	0:12	0:04	0:05	0*	0*	0*	0:10	0:51	0:27	0*	0:00	0*
1:96%	***CVaR***	0:8%	0:8%	6:5%	0:6	0:57	0*	0:3	0:3	0*	0:9	0:9	0:5
	CVaR	4:1%	4:1%	7:3%	0*	0:26	0*	0:14	0:14	0*	0:13	0:51	0:26
0:38%	***CVaR***	0%	0:8%	3:2%	***NA***	0:78	0*	0:33	0:5	0*	***NA***	0:9	0:1
	CVaR	1:6%	2:4%	4:1%	0:23	0:04	0*	0:1	0:01	0*	0:8	0:7	0:5
				:							:92 6	:0181	0: CV aR 3:
1:96%	***CVaR***	1:6%											

Stage 2 *E¢ ciency measures.*

S

“non-superiority”

S

“non-superiority” **S**

“non-superiority”

S *“non-superiority”*

*

SVL vs SVJL

SVL vs SVJ

SVL vs SVJ

Out of sample RLF and FLF loss function approach applied to the models surviving the VaR Backtesting stage.

	5%		1%		5%		1%	
	0	0:03	[illegible]	[illegible]	0:01	0:02	0	0:01
	0	0:07	0	0:11	0:01	0:01	0	0:01

Out of sample RLF and FLF loss function approach applied to the models surviving the CVaR Backtesting stage

References

Appendix I: Realized Variance and Moment conditions

Realized Variance *Integrated Variance*

8.1 No jumps

$$r(t;k) = \ln F_{t} \quad \ln F_{-} = \int_{-}^{[t \quad k;t]} (\ )\, d + \int_{-} (\ )\, dW$$

$$QV(t;k) = IV(t;k) = \int_{-} {}^{2}(\ )\, d$$

Realized Variance

$$RV(t;k;n) = \sum_{=1} r\left(t \quad k + \frac{j}{n}; \frac{1}{n}\right)^{2}$$

$$RV(t;k;n) \rightarrow IV(t;k)$$

$$n \ ! \ 1$$

$I = {}^{1}$

8.2 Jumps

t $[t \quad k; t]$

$$
\begin{aligned}
r(t;k) &= \ln F \quad \ln F_{-} \\
&= \int_{-} (\ ) d \ + \int_{-} (\ ) dW \ + \int_{-} x(\ ) dN(\ )
\end{aligned}
$$

$$e_1 = E[BP_{+1} j G] + {}^2 dt \quad RV_{+1}$$

$$E[RV_{+1} j G] = E[BP_{+1} j G] + {}^2 dt$$

$(A{:}3)$ *Appendix A.1*

Residual 2

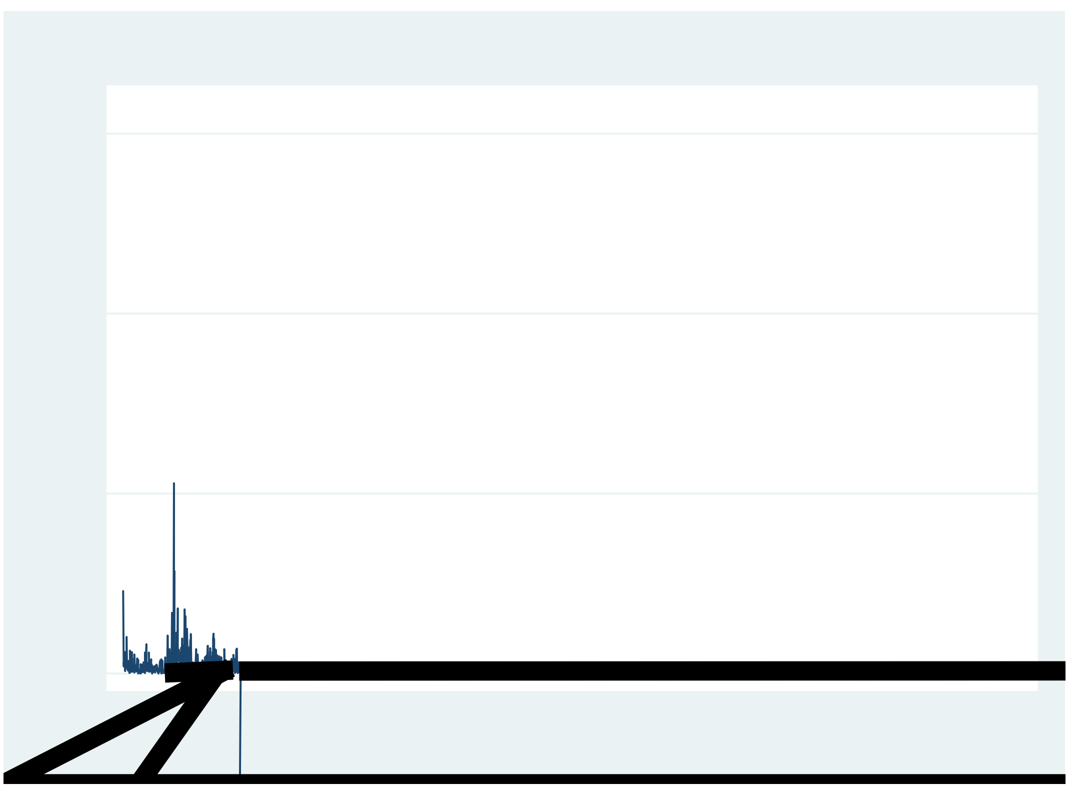
$(t + 1; t + 2)$

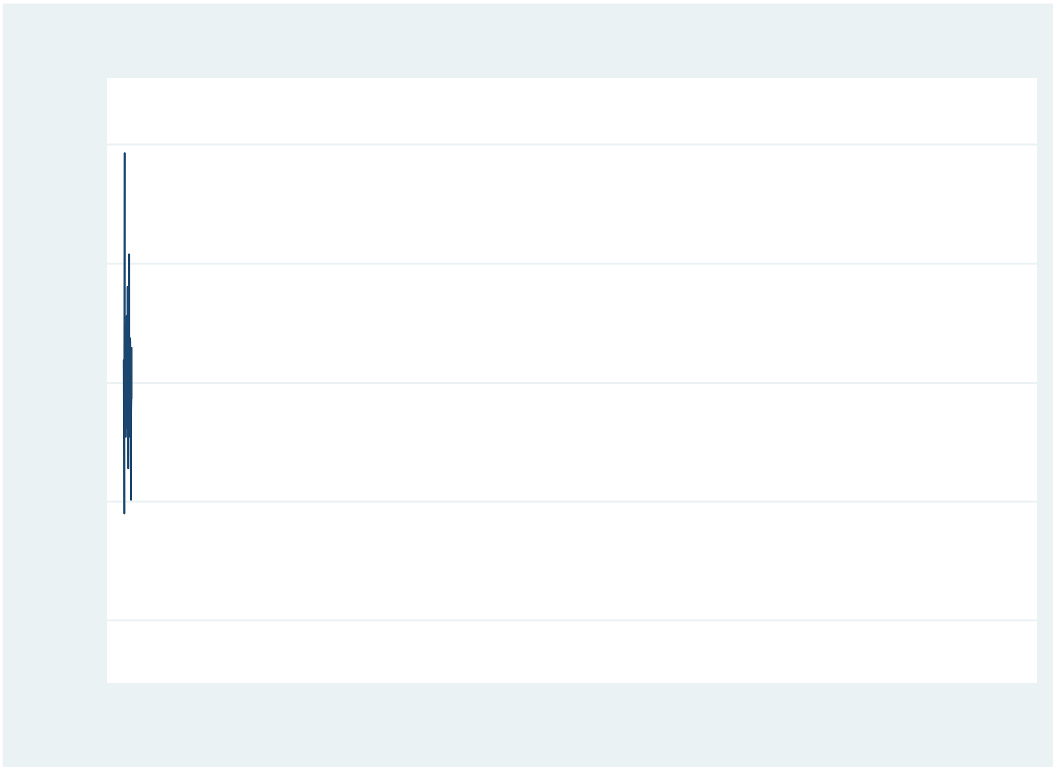
$$e_2 = E[RV_{+1}$$

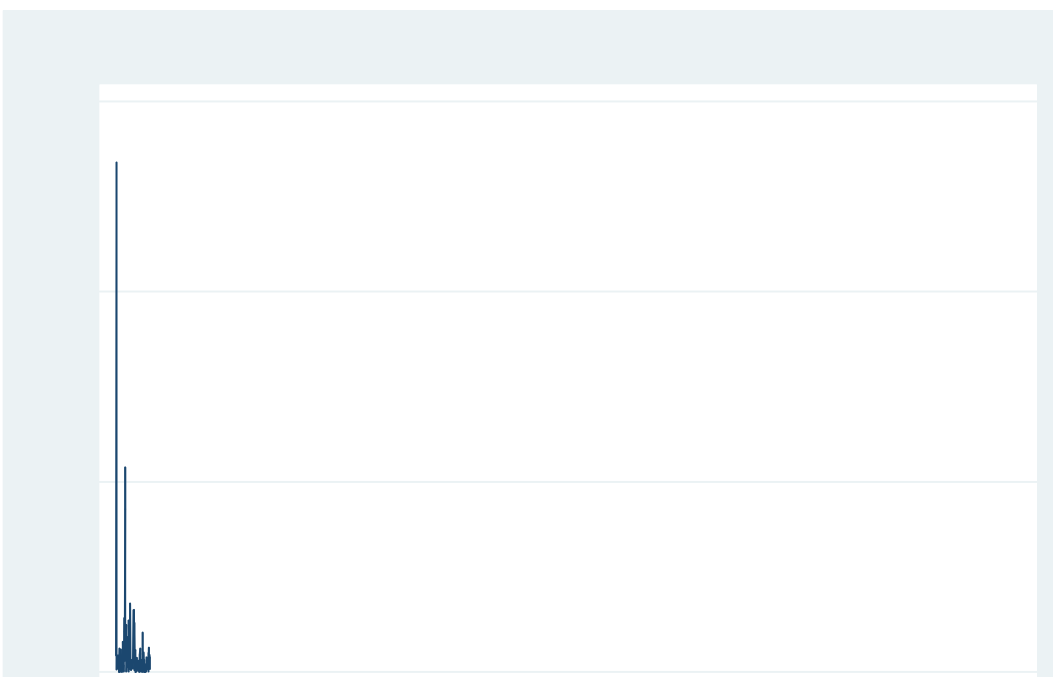
$\boldsymbol{E}$ $\boldsymbol{V}^{2}{}_{+1}$ G

$\boldsymbol{V}^{2}_{+1}$ $_{+2}$

Appendix II: Figures







Appendix III: t and J tests on the moment conditions

GMM estimates for SV model for the S&P500 futures: 09/2001–06/2016

$E[V_{+1\ +2} \mid G]\quad V_{+1\ +2}$	0:0052	0:00271	0:997841
$E[V_{+1\ +2} V_{-16\ -15} \mid G]\quad V_{+1\ +2} V_{-16\ -15}$	0:0052	0:00844	0:993267
$E[V_{+1\ +2} V_{-17\ -16} \mid G]\quad V_{+1\ +2} V_{-17\ -16}$	0:0052	0:05157	0:958872
$E\ V^2_{+1\ +2}\ G\quad V^2_{+1\ +2}$	0:00851	0:00735	0:994135
$E\ V^2_{+1\ +2} V^2_{-10\ -9}\ G\quad V^2_{+1\ +2} V^2_{-10\ -9}$	0:00851	0:01141	0:9909
$E\ V^2_{+1\ +2} V^{-}$			

GMM estimates for SVJ model for the S&P500 futures: 09/2001–06/2016

$E[V_{+1\ \ +2} \mid G]\ \ V_{+1\ \ +2}$	$0{:}000068$	$0{:}999994$
$E[V_{+1\ \ +2}V_{-16\ \ -15} \mid G]\ \ V_{+1\ \ +2}V_{-16\ \ -15}$	$0{:}000068$	$0{:}999995$
$E[V_{+1\ \ +2}V_{-17\ \ -16} \mid G]\ \ V_{+1\ \ +2}V_{-17\ \ -16}$	$0{:}000068$	$0{:}999950$
$E\ \ V^2_{+1\ \ +2}\ G\ \ V^2_{+1\ \ +2}$	$0{:}000865$	$0{:}999991$
$E\ \ V^2_{+1\ \ +2}V^2_{-10\ \ -9}\ G\ \ V^2_{+1\ \ +2}V^2_{-10\ \ -9}$	$0{:}000865$	$0{:}999930$
$E\ \ V^2_{+1\ \ +2}V^2_{-18\ \ -17}\ G\ \ V^2_{+1\ \ +2}V^2_{-18\ \ -17}$	$0{:}000865$	$0{:}999938$
$E[V_{+1} \mid G]\ \ V_{+1}$	$0{:}002765$	$0{:}999498$
$E\ \ V_{+1}BP^2_{-16\ \ -15}\ G\ \ V_{+1}BP^2_{-16\ \ -15}$	$0{:}002765$	$0{:}999712$
$E\ \ V_{+1}BP^2_{-2\ \ -1}\ G\ \ V_{+1}BP^2_{-2\ \ -1}$	$0{:}002765$	$0{:}993492$
$E\ \ V^2_{+1}\ G\ \ V^2_{+1}$	$0{:}000559$	$0{:}999998$
$E\ \ V^2_{+1}V^2_{-10\ \ -9}\ G\ \ V^2_{+1}V^2_{-10\ \ -9}$	$0{:}000559$	$0{:}999986$
$E\ \ V^2_{+1}BP^2_{-18\ \ -17}\ G\ \ V^2_{+1}BP^2_{-18\ \ -17}$	$0{:}000559$	$0{:}999803$
J	$^2_7 = 6{:}96664$	$0{:}4324$

p J

GMM estimates for SVL model for the S&P500 futures: 09/2001–06/2016

$E[V_{+1}\ {}_{+2}\mid G]\quad V_{+1}\ {}_{+2}$	0.0001	0	1
$E[V_{+1}\ {}_{+2}V_{-16}\ {}_{-15}\mid G]\quad V_{+1}\ {}_{+2}V_{-16}\ {}_{-15}$	0.0001	0	1
$E[V_{+1}\ {}_{+2}V_{-17}\ {}_{-16}\mid G]\quad V_{+1}\ {}_{+2}V_{-17}\ {}_{-16}$	0.0001	0	1
$E[V_{+1}\ {}_{+2}V_{-20}\ {}_{-19}\mid G]\quad V_{+1}\ {}_{+2}V_{-20}\ {}_{-19}$	0.0001	0.000025	0.99998
$E\ V^2_{+1}\ {}_{+2}\ G\quad V^2_{+1}\ {}_{+2}$	0.000232	0	1
$E\ V^2_{+1}\ {}_{+2}V^2_{-20}\ {}_{-19}\ G\quad V^2_{+1}\ {}_{+2}V^2_{-20}\ {}_{-19}$	0.000232	0	1
$E\ V^2_{+1}\ {}_{+2}V^2_{-18}\ {}_{-17}\ G\quad V^2_{+1}\ {}_{+2}V^2_{-18}\ {}_{-17}$	0.000232	0	1
$E\ V^2_{+1}\ {}_{+2}V^2_{-22}\ {}_{-21}\ G\quad V^2_{+1}\ {}_{+2}V^2_{-22}\ {}_{-21}$	0.000232	0.000025	0.99998
$E[p\ {}_{+1}V_{+1}\ {}_{+2}\mid G]\quad p\ {}_{+1}V_{+1}\ {}_{+2}$	0.00023	0	1
$E\ p\ {}_{+1}V_{+1}\ {}_{+2}p_{-15}\ {}_{-14}V^2_{-15}\ {}_{-14}\ G$ $p\ {}_{+1}V_{+1}\ {}_{+2}p_{-15}\ {}_{-14}V^2_{-15}\ {}_{-14}$	0.00023	0	1
$E\ p\ {}_{+1}V_{+1}\ {}_{+2}p_{-12}\ {}_{-11}V^2_{-12}\ {}_{-11}\ G$ $p\ {}_{+1}V_{+1}\ {}_{+2}p_{-12}\ {}_{-11}V^2_{-12}\ {}_{-11}$	0.00023	0	1
$E\ p\ {}_{+1}V_{+1}\ {}_{+2}p_{-18}\ {}_{-17}V^2_{-18}\ {}_{-17}\ G$ $p\ {}_{+1}V_{+1}\ {}_{+2}p_{-18}\ {}_{-17}V^2_{-18}\ {}_{-17}$	0.00023	2.9E 05	0.999977
J	$^2_8$	11.177	0.1919

t p J

GMM estimates for SVJL model for the S&P500 futures: 09/2001–06/2016

E [V +1 +2 j G] V +1 +2	2:2E 05		0	1
E V +1 +2 V² −20 −19 G V +1 +2 V² −20 −19	2:2E 05		0	1
E V +1 +2 V³ −18 −17 G V +1 +2 V³ −18 −17	2:2E 05	2E 06		0:999998
E V² +1 +2 G V² +1 +2	3E 06		0	1
E V² +1 +2 V² −10 −9 G V² +1 +2 V² −10 −9	3E 06		0	1
E V² +1 +2 V⁴ −25 −24 G V² +1 +2 V⁴ −25 −24	3E 06		0	1
E [p +1 V +1 +2 j G] p +1 V +1 +2	3E 06		0	1
E [p +1 V +1 +2 p −15 −14 V −15 −14 j G] p +1 V +1 +2 p −15 −14 V −15 −14	3E 06		0	1
E [p +1 V +1 +2 p −12 −11 V −12 −11 j G] p +1 V +1 +2 p −12 −11 V −12 −11	3E 06		0	1
E [V +1 j G] V +1	0:000021		0	1
E V +1 V¯ −16 −15 G V +1 V¯ −16 −15	0:000021		0	1
E V +1 V⁴ −22 −21 G V +1 V⁴ −22 −21	0:000021		0	1
E V² +1 G V² +1	3E 06		0	1
E V² +1 V² −10 −9 G V² +1 V² −10 −9	3E 06		0	t 2 1
E V² +1 V −18 −17 G V² +1 V −18 −17	3E 06 V +1 +1		12 V 0	1

GMM estimates for SVJ model for the Natural Gas futures: 09/2001–06/2016

$E[V_{+1} \; {}_{+2} \mid G] \quad V_{+1} \; {}_{+2}$	0:005914	0:999994
$E[V_{+1} \; {}_{+2} V_{-16} \; {}_{-15} \mid G] \quad V_{+1} \; {}_{+2} V_{-16} \; {}_{-15}$	0:005914	0:999995
$E[V_{+1} \; {}_{+2} V_{-17} \; {}_{-16} \mid G] \quad V_{+1} \; {}_{+2} V_{-17} \; {}_{-16}$	0:005914	0:999950
$E \; V^2_{+1} \; {}_{+2} \; G \quad V^2_{+1} \; {}_{+2}$	0:009844	0:999991
$E \; V^2_{+1} \; {}_{+2} V^2_{-10} \; {}_{-9} \; G \quad V^2_{+1} \; {}_{+2} V^2_{-10} \; {}_{-9}$	0:009844	0:999930
$E \; V^2_{+1} \; {}_{+2} V^2_{-18} \; {}_{-17} \; G \quad V^2_{+1} \; {}_{+2} V^2_{-18} \; {}_{-17}$	0:009844	0:999938
$E[V \; {}_{+1} \mid G] \quad V \; {}_{+1}$	0:002155	0:999498
$E \; V \; {}_{+1} BP^2_{-16} \; {}_{-15} \; G \quad V \; {}_{+1} BP^2_{-16} \; {}_{-15}$	0:002155	0:999712
$E \; V \; {}_{+1} BP^2_{-2} \; {}_{-1} \; G \quad V \; {}_{+1} BP^2_{-2} \; {}_{-1}$	0:002155	0:993492
$E \; V^2 \; {}_{+1} \; G \quad V^2 \; {}_{+1}$	0:005225	0:999998
$E \; V^2 \; {}_{+1} V^2_{-10} \; {}_{-9} \; G \quad V^2 \; {}_{+1} V^2_{-10} \; {}_{-9}$	0:005225	0:999986
$E \; V^2 \; {}_{+1} BP^2_{-18} \; {}_{-17} \; G \quad V^2 \; {}_{+1} BP^2_{-18} \; {}_{-17}$	0:005225	0:999803
J	$^2_7 = 5{:}81155$	0:5619

$p \qquad J$

GMM estimates for SVL model for the Natural Gas futures: 09/2001–06/2016

$E[V_{+1} \; {}_{+2} j G] \quad V_{+1} \; {}_{+2}$	0:0058	0:00262	0:997913
$E[V_{+1} \; {}_{+2} V_{-16} \; {}_{-15} j G] \quad V_{+1} \; {}_{+2} V_{-16} \; {}_{-15}$	0:0058	0:00277	0:997792
$E[V_{+1} \; {}_{+2} V_{-17} \; {}_{-16} j G] \quad V_{+1} \; {}_{+2} V_{-17} \; {}_{-16}$	0:0058	0:0678	0:945947
$E \; V^2_{+1} \; {}_{+2} \; G \quad V^2_{+1} \; {}_{+2}$	0:01161	0:01107	0:991166
$E \; V^2_{+1} \; {}_{+2} V^2_{-10} \; {}_{-9} \; G \quad V^2_{+1} \; {}_{+2} V^2_{-10} \; {}_{-9}$	0:01161	0:0099	0:992101
$E \; V^2_{+1} \; {}_{+2} V^2_{-18} \; {}_{-17} \; G \quad V^2_{+1} \; {}_{+2} V^2_{-18} \; {}_{-17}$	0:01161	0:18512	0:853143
$E[p \; {}_{+1} V_{+1} \; {}_{+2} j G] \quad p \; {}_{+1} V_{+1} \; {}_{+2}$	0:00281	0:00116	0:999072
$E[p \; {}_{+1} V_{+1} \; {}_{+2} p_{-15} \; {}_{-14} V_{-15} \; {}_{-14} j G]$ $p \; {}_{+1} V_{+1} \; {}_{+2} p_{-15} \; {}_{-14} V_{-15} \; {}_{-14}$	0:00281	0:00544	0:995658
$E[p \; {}_{+1} V_{+1} \; {}_{+2} p_{-18} \; {}_{-17} V_{-18} \; {}_{-17} j G]$ $p \; {}_{+1} V_{+1} \; {}_{+2} p_{-18} \; {}_{-17} V_{-18} \; {}_{-17}$	0:00281	0:02369	0:9811
J	$^2_5$		

GMM estimates for SVJL model for the Natural Gas futures: 09/2001–06/2016

$E[V_{+1} \; {}_{+2} \mathrm{j} G] \quad V_{+1} \; {}_{+2}$	0:00061	0:00029	0:999771
$E[V_{+1} \; {}_{+2} V_{-10} \; {}_{-9} \mathrm{j} G] \quad V_{+1} \; {}_{+2} V_{-10} \; {}_{-9}$	0:00061	0:00194	0:998452
$E \; V_{+1} \; {}_{+2} V^{3}_{-18} \; {}_{-17} \; G \quad V_{+1} \; {}_{+2} V^{3}_{-18} \; {}_{-17}$	0:00061	0:00024	0:999809
$E \; V^{2}_{+1} \; {}_{+2} \; G \quad V^{2}_{+1} \; {}_{+2}$	0:00602	0:00574	0:995417
$E \; V^{2}_{+1} \; {}_{+2} V^{2}_{-10} \; {}_{-9} \; G \quad V^{2}_{+1} \; {}_{+2} V^{2}_{-10} \; {}_{-9}$	0:00602	0:14815	0:882232
$E \; V^{2}_{+1} \; {}_{+2} V^{4}_{-25} \; {}_{-24} \; G \quad V^{2}_{+1} \; {}_{+2} V^{4}_{-25} \; {}_{-24}$	0:00602	0:01217	0:99029
$E[p \; {}_{+1} V_{+1} \; {}_{+2} \mathrm{j} G] \quad p \; {}_{+1} V_{+1} \; {}_{+2}$	0:000105	0:00003	0:999976
$E[p \; {}_{+1} V_{+1} \; {}_{+2} p_{-15} \; {}_{-14} V_{-15} \; {}_{-14} \mathrm{j} G]$ $p \; {}_{+1} V_{+1} \; {}_{+2} p_{-15} \; {}_{-14} V_{-15} \; {}_{-14}$	0:000105	0:000015	0:999988
$E[p \; {}_{+1} V_{+1} \; {}_{+2} p_{-12} \; {}_{-11} V_{-12} \; {}_{-11} \mathrm{j} G]$ $p \; {}_{+1} V_{+1} \; {}_{+2} p_{-12} \; {}_{-11} V_{-12} \; {}_{-11}$	0:000105	0:000264	0:999789
$E[V \; {}_{+1} \mathrm{j} G] \quad V \; {}_{+1}$	0:052523	0:057867	0:953858
$E \; V \; {}_{+1} V^{-}_{-6} \; {}_{-5} \; G \quad V \; {}_{+1} V^{-}_{-6} \; {}_{-5}$	0:052523	4:326995	0:000016
$E \; V \; {}_{+1} V^{4}_{-2} \; {}_{-1} \; G \quad V \; {}_{+1} V^{4}_{-2} \; {}_{-1}$	0:052523	0:344424	0:730547
$E \; V^{2}{}_{+1} \; G \quad V^{2}{}_{+1}$	0:00536	0:00157	0:998747
$E \; V^{2}{}_{+1} V^{2}_{-10} \; {}_{-9} \; G \quad V^{2}{}_{+1} V^{2}_{-10} \; {}_{-9}$	0:00536	0:00151	0:998795
$E \; V^{2}{}_{+1} V_{-18} \; {}_{-17} \; G \quad V^{2}{}_{+1} V_{-18} \; {}_{-17}$	0:00536	0:00746	0:994051
J		$^{2}_{9} = 6{:}00869$	0:7390

$p \quad J$

GMM estimates for SV model for the WTI futures: 09/2001–06/2016

$E[V_{+1\ +2} \mid G]\quad V_{+1\ +2}$	0:000451	0:000079	0:999937
$E[V_{+1\ +2} V_{-16\ -15} \mid G]\quad V_{+1\ +2} V_{-16\ -15}$	0:000451	0:000044	0:999965
$E[V_{+1\ +2} V_{-17\ -16} \mid G]\quad V_{+1\ +2} V_{-17\ -16}$	0:000451	0:000586	0:999532
$E\ V_{+1\ +2}$			

GMM estimates for SVL model for the WTI futures: 09/2001–06/2016

$E[V_{+1} \; {}_{+2} \mid G] \quad V_{+1} \; {}_{+2}$	0:000451	0:000017	0:999987
$E[V_{+1} \; {}_{+2} V_{-19} \; {}_{-18} \mid G] \quad V_{+1} \; {}_{+2} V_{-19} \; {}_{-18}$	0:000451	0:00002	0:999984
$E[V_{+1} \; {}_{+2} V_{-17} \; {}_{-16} \mid G] \quad V_{+1} \; {}_{+2} V_{-17} \; {}_{-16}$	0:000451	0:000548	0:999563
$E \; V^2_{+1} \; {}_{+2} \; G \quad V^2_{+1} \; {}_{+2}$	0:001539	0:000007	0:999994
$E \; V^2_{+1} \; {}_{+2} V^2_{-10} \; {}_{-9} \; G \quad V^2_{+1} \; {}_{+2} V^2_{-10} \; {}_{-9}$	0:001539	0:000024	0:999981
$E \; V^2_{+1} \; {}_{+2} V^2_{-18} \; {}_{-17} \; G \quad V^2_{+1} \; {}_{+2} V^2_{-18} \; {}_{-17}$	0:001539	0:001058	0:999156
$E[p \; {}_{+1} V_{+1} \; {}_{+2} \mid G] \quad p \; {}_{+1} V_{+1} \; {}_{+2}$	0:000108	0:000006	0:999995
$E[p \; {}_{+1} V_{+1} \; {}_{+2} p_{-18} \; {}_{-17} V_{-18} \; {}_{-17} \mid G]$ $p \; {}_{+1} V_{+1} \; {}_{+2} p_{-18} \; {}_{-17} V_{-18} \; {}_{-17}$	0:000108	0:000001	0:999999
$E[p \; {}_{+1} V_{+1} \; {}_{+2} p_{-16} \; {}_{-15} V_{-16} \; {}_{-15} \mid G]$ $p \; {}_{+1} V_{+1} \; {}_{+2} p_{-16} \; {}_{-15} V_{-16} \; {}_{-15}$	0:000108	0:000097	0:999923
J	$^2_5$	1:37354	0:9272

t p J