

$\frac{1}{2} q^2$ .

**Exercise 2.** Let  $R$  be an integral domain that is integrally closed in its field of fraction  $F$ .

- (1) Show that an algebraic element is integral over  $R$  if and only if its minimal polynomial over  $F$  is a monic polynomial in  $R[x]$ .
- (2) Show that for any monic  $f(x) \in R[x]$ , for any decomposition  $f(x) = f_1(x)f_2(x)$  into monic polynomials in  $F[x]$ , the factors  $f_1, f_2$  have coefficients in  $R$ .

**Exercise 3.** Let  $k$  be an algebraically closed field. Consider the affine variety  $V = k^2$  (with coordinates  $x, y$ ), and the affine variety  $W = k^2$  (with coordinates  $s, t$ ). Suppose  $\phi : V \rightarrow W$  is a morphism, and denote by  $R = k[x, y]$  the image of the induced ring homomorphism  $\phi^* : k[s, t] \rightarrow k[x, y]$ . For each of the following statements, give a proof or a counterexample.

- (1) If  $\phi$  has Zariski dense image, then  $\phi^*$  is surjective.
- (2) If  $k[x, y] = R$  is any integral extension of  $k[s, t]$ , then  $\phi^*$  is surjective.

**Exercise 7.** Suppose  $k$  be a field and  $R = k[x; y; z]$  a polynomial ring. Compute

$$\text{Ext}_R^i(R=(xz); R=(xy; xz))$$

for all  $i \geq 0$ .

**Exercise 8.** Suppose  $p$  is a prime of the form  $4k + 3$ . Find the conjugacy class of every element of order 4 in  $\text{GL}_2(\mathbb{F}_p)$ .