ANALYSIS QUALIFYING EXAM SYLLABUS, 2013

Schwarz's lemma, residues, the Residue Theorem, the Argument Principle, Rouche's theorem, use of residues to evaluate definite integrals.

- Further properties of harmonic functions: the mean-value property, Poisson's formula, Schwarz's Theorem on harmonic functions, Schwarz's reflection principle for harmonic functions and for analytic functions.
- Weierstrass's Theorem on a series of analytic functions, Hurwitz's theorem on nonvanishing of a limit function, Taylor Series, Laurent Series.
- Partial fraction representation of meromorphic functions on  $\mathbb{C}$  with given principle parts and examples, infinite product representation of entire functions with given zeros and examples, the Gamma function and its basic properties, Stirling's formula.
- Equicontinuity, normal families of functions, Arzela's theorem, normal families of analytic functions characterized by local boundedness.
- The Riemann mapping theorem and its proof, behavior of the mapping at the boundary,