Algebra Qualifying Exam, Fall 2013 You have 3 hours to answer all problems.

1. Classify, up to isomorphism, all groups of order 385 = 57 11.

2. Determine the Galois group of the polynomial X^5 2 2 Q[X].

3. Let *R* be a local ring with maximal ideal M. Suppose that $f : A \mid B$ is a homomorphism of nitely generated free *R*-modules with the property that the induced map $A=MA \mid B=MB$ is an isomorphism Show that *f* is itself an isomorphism.

4. The ring of integers of $Q[^{p}\overline{7}]$ is $Z[^{p}\overline{7}]$. For each of the following primes $p \ge Z$, describe how the ideal $pZ[^{p}\overline{7}]$ factors as a product of prime ideals (\describe" means give the number of prime factors, their multiplicities in the factorization, and the cardinalities of the residue elds):

- (a) p = 2
- (b) p = 7
- (c) p = 17.

5. Let A be an n matrix with entries in an algebraically closed eld. Show that A is similar to a diagonal matrix if and only if the minimal polynomial of A has no repeated roots.

6. Let *R* be a commutative ring with 1, *N* an *R*-module, and for every maximal idealm *R* let $N_{\rm m}$ be the localization of *N* at m. Prove that the natural map $N \neq \frac{1}{2} N_{\rm m}$ is injective.

- 7. Let *k* be a eld, R = k[x; y] and I = (x; y).
- (a) Prove that / is neither at nor projective as an *R*-module.
- (b) Compute $\operatorname{Ext}_{R}^{1}(R=I;I)$.

8. Let *k* be an algebraically closed eld. Consider the a ne variety $V = k^2$ with coordinates *x*; *y*, and the a ne variety $W = k^2$ with coordinates *s*; *t*. Suppose *f*: $V \neq W$ a morphism, and denote by *R* the image of the induced pull-back map *f* : $k[s; t] \neq k[x; y]$. For each of the following statements, give a proof or a counterexample.

- (a) If *f* has Zariski dense image, then *f* is surjective.
- (b) If k[x; y]=R is an integral extension of rings, then f is surjective.