Algebra qualifying exam, Spring 2016

1. Classify the groups of order 24 having trivial center.

2. Let V be a finite dimensional vector space over a field F of characteristic p and let $T : V \to V$ be a linear transformation such that $T^p = I$

$\overline{d}$) for some $d \in \mathbb{Z}$.

5. Suppose n and m are positive integers, let $R = \mathbb{Z}[X]/(X^n)$ and let M be an R-module, let x be the image of X in R, and let (x^m) be the ideal in R generated by x^m. Compute $\mathrm{Tor}_i^R(M, (x^m))$ for all i.

6. Let k be a field. Find the minimal primes and compute the Krull dimension of $R = k[x, y, z]/(xy, xz)$.

7. Let R be an Artinian local ring. Prove that an R-module is flat if and only if it is free.

8. Suppose that R is a Noetherian ring and $\mathfrak{p} \subset R$ is a prime ideal such that $R_{\mathfrak{p}}$ is an integral domain. Show that there is an $f \notin \mathfrak{p}$ such that R_f is an integral domain. (Recall that $R_f = S^{-1}R$ where $S = \{1, f, f^2, f^3, \ldots\}$.)